HOLY TRINITY INTERNATIONAL SCHOOL

Mathematics

Grade X

1. Prove that $\sqrt{2}$ is an irrational number.
   Solution:
   Let's assume that $\sqrt{2}$ is a rational number. Let $\sqrt{2} = \therefore \frac{a}{b}$, where a and b are co primes
   [$\frac{a}{b}$ is the general form of a rational number]
   $(\sqrt{2})^2 = (\frac{a}{b})^2$ [squaring on both sides]
   $2=\frac{a^2}{b^2}$
   $2b^2 = a^2$ .... (1) { $\therefore$ cross multiplication}
   $\therefore$ 2 is a factor of a^2
   Then 2 is also a factor of a.
   Let's assume that a = 2 c in equation (i)
   $\therefore 2b^2 = (2c)^2$
   $2b^2 = 4c^2$
   $b^2 = 2c^2$
   $\therefore$ 2 is a factor of b^2
   $\therefore$ 2 is also a factor of b.
   Then we get 2 *is a common* factor of both a and b. This contradicts the fact that a and b are co-primes.
   $\therefore$ [co-primes have no common factor other than 1]
   $\therefore$ our assumption is wrong.
   Then we conclude that $\sqrt{2}$ is an irrational number.

2. Prove that $\sqrt{3}$ is an irrational number. solution:
   Let's assume that $\sqrt{3}$ is a rational number. Let $\sqrt{3} = \frac{a}{b}$ where a and b are coprimes.
   $(\sqrt{3})^2 = \frac{a^2}{b^2}$
   $3 = \frac{a^2}{b^2}$
   $3b^2 = a^2$ ....(1)
   $\therefore$ 3 *is a factor* of a^2
   $\therefore$ Then 3 is also a factor of a.
   Let $a = 3c$. in equa (1)
   $3b^2 = (3c)^2$
   $3b^2 = 9c^2$
   $b^2 = 3c^2$
   $\therefore$ 3 is a factor of b^2
   Then 3 will be a factor of b.
   $\therefore$3 will be a common factor of both a and b. This contradicts the fact that a and b are coprimes. $\therefore$ Our assumption is wrong.
   Now we prove that $\sqrt{3}$ is an irrational number.

Home work

1. Prove that $\sqrt{5}$ is an irrational number.
2. Prove that $\sqrt{7}$ is an irrational number.