

HOLY TRINITY INTERNATIONAL SCHOOL

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Grade X

Mathematics

Chapter : 2 – Polynomials

A. Linear Polynomials.

What is a linear Polynomial? A Polynomial has the highest degree one is called a linear polynomial.

Examples : $3x+1$, $2x-5$, $x+2$, $\frac{1}{x+7}$, $x-\frac{12}{13}$, ... Here the degree of 'x' is one.

B. Quadratic Polynomial.

What is a quadratic Polynomial? A Polynomial of degree two is called a quadratic polynomial. There are two or three terms.

Examples : x^2-5x+6 , x^2-9 , $2x^2+5x-\frac{3}{2}$.

$$\frac{x}{2} - \frac{3x^2}{5} + 7, \sqrt{3}y^2 + 2y - 1, \dots$$

C. What is the general form of a quadratic polynomial?

ax^2+bx+c , Where 'a' is the co-efficient of x^2 , 'b' is the coefficient of x and 'c' is the constant. ie., a, b and c are real numbers and $a \neq 0$.

D. What is a cubic polynomial?

A polynomial of degree 3 is called a cubic polynomial.

Examples : $9x^3 - x^2 + 5x+1$, x^3-1 , $2x^3+5x$, x^3-x^2+x+1 , $7x^3+x^2-3$, ...

E. What is the general form of cubic polynomial?

ax^3+bx^2+cx+d , where a, b, c and d are real numbers, and $a \neq 0$.

F. Zero of a linear polynomial.

What is the general form of a linear polynomial?

$ax + b$ (is the answer) where 'a' is the coefficient of x and b is the constant.

Usually a polynomial is denoted by P(x), if K is a zero of $P(x)=ax+b$, then

$$P(k)=ak + b=0, ak=-b, \text{ then } K = -\frac{b}{a}.$$

There fore, the zero of a linear polynomial $ax + b$ is $-\frac{b}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x}$.

1. Example. Find the zero of the following linear polynomials

- (i) $3x-5$ (ii) $2x+3$ (iii) $x-2$ (iv) $5x-1$

Study the formula for find the zero of a linear polynomial ($-\frac{b}{a}$).

(i) $3x - 5$

zero is $-\frac{b}{a}$, $b = -5$, and $a = 3$.

$$\text{zero} = -\frac{-5}{3} = \frac{5}{3} \quad [-x = +]$$

(ii) $2x+3$

zero = $-\frac{b}{a}$; $b=3$, $a=2$

$$= -\frac{3}{2}$$

$$(iii) \quad x-2; \text{ zero} = -\frac{b}{a}, \quad b = -2 \text{ and } a = 1$$

$$= \frac{-(-2)}{1} = \frac{2}{1} = 2$$

$$(iv) \quad 5x-1; \text{ zero} = -\frac{b}{a}; \quad b = -1, a = 5$$

$$= \frac{-(-1)}{5} = \frac{1}{5}$$

Example 2. Find zero of the following polynomials.

1. $x^2 - 5x + 6$

$x = 2$, we get $x^2 - 5x + 6 = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6 = 10 - 10 = 0$;

Put $x = 3$; $x^2 - 5x + 6 = (3)^2 - 5(3) + 6$

$= 9 - 15 + 6 = 15 - 15 = 0$

2 and 3 are the zeroes of the polynomial $x^2 - 5x + 6$.

2. $3x^2 + 2x - 5$

Solution: Put $x = 1$, hence we get $3x^2 + 2x - 5$

$= 3(1)^2 + 2(1) - 5 = 3(1) + 2 - 5$

$= 3 + 2 - 5 = 5 - 5 = 0$.

then put $x = \frac{-5}{3}$, then $3x^2 + 2x - 5$ is

$3\left(\frac{-5}{3}\right)^2 + 2\left(\frac{-5}{3}\right) - 5$

$3\left(\frac{25}{9}\right) + \left(\frac{-10}{3}\right) - 5$

$\left(\frac{25}{3}\right) + \left(\frac{-10}{3}\right) - 5 = \frac{25 - 10 - 15}{3} = \frac{25 - 25}{3} = 0$; LCM = 3

1 and $\frac{-5}{3}$ are called the zeros of $3x^2 + 2x - 5$.

Home Work:

1. Find the zeroes of the following.

(i) $2x - 6$ (ii) $3x - 1$ (iii) $5x - 5$ (iv) $2x + 3$

2. Find the zeroes of the following.

(i) $p(x) = x^2 + 5x + 6$

(ii) $p(x) = x^2 + x - 2$

(iii) $x^3 + 3x^2 + 4x + 2$

(iv) if 3 is a zero of the polynomial $2x^2 + x + k$, then the value of $k =$ _____

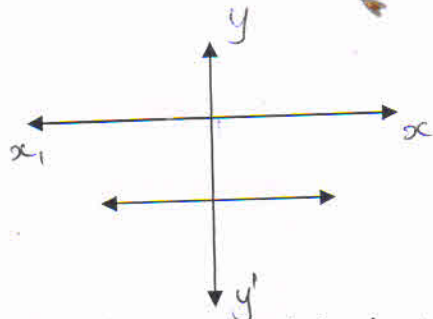
Find the zeros. (Study the definition)

The intersection of the graph of a polynomial with 'x' axis is known as zero of the polynomial.

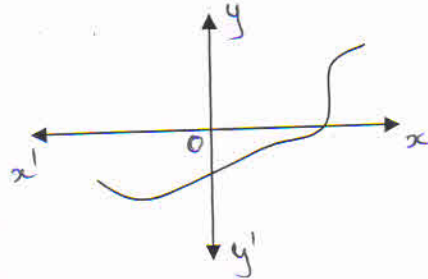
1. The graphs of $y = p(x)$ are given. Find the number of zeroes of each case.

Ans:

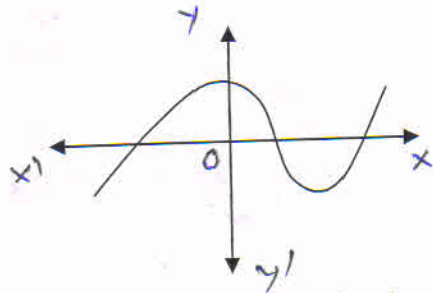
(i) This graph does not intersect the 'x' axis. Therefore this graph $p(x)$ has no zero.



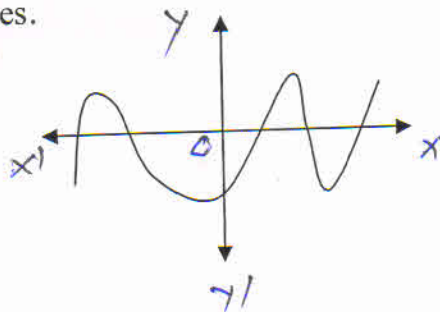
- (ii) This graph intersects the 'x' axis at any one point. It has only one zero.



- (iii) This graph intersects the 'x' axis at 3 points. This graph has 3 zeroes.



- (iv) This graph intersects the 'x' axis at 5 points. Therefore this graph has 5 zeroes.



2. Find the zeroes of the quadratic polynomial $x^2+(a+1)x+b$ are 2 and -3 then find the values of a and b.

$$\text{Let } p(x) = x^2+(a+1)x+b$$

$$\text{if 2 is a zero of } p(x), \quad p(2)=0$$

$$p(2) = x^2+(a+1)x+b=0$$

$$(2)^2+(a+1)(2)+b=0$$

$$4+2a+2+b=0$$

$$2a+b+6=0$$

$$2a+b = -6 \quad \text{_____} (1)$$

$$\text{if -3 is a zero of } p(x)$$

$$p(-3)=0$$

$$p(-3)=(-3)^2+(a+1)(-3)+b=0$$

$$9-3a-3+b=0$$

$$6-3a+b=0$$

$$-3a+b=-6 \quad (2)$$

(solve using elimination method)

$$\text{Equation (1) } 2a+b=-6$$

$$(2) -3a+b=-6$$

$$\text{Subtract } 5a=0 \quad \text{So } a=0$$

$$\text{put } a=0 \text{ in equation (1) } = 2a+b=-6$$

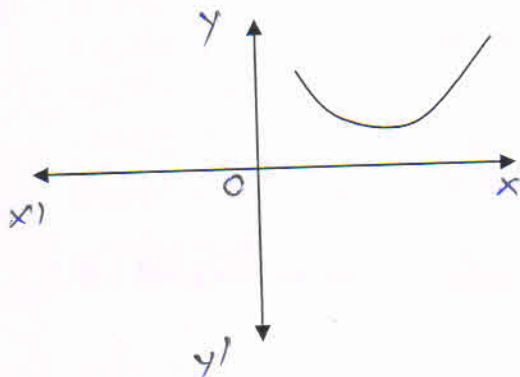
$$0+b=-6;$$

$$b=-6$$

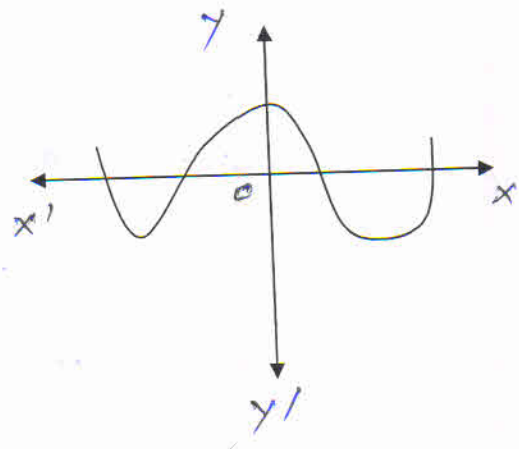
$$a=0, b=-6$$

Home work. Find the zeroes.

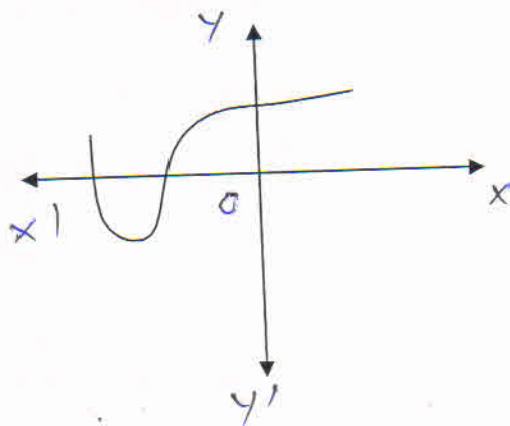
(i)



(iii)



(ii)



Relationship between zeroes and coefficients of a polynomial.

NB: Study these important points first, then try to understand the solutions.

(i) Quadratic polynomial : ax^2+bx+c .

(ii) If α and β are the zeroes.

$$\text{Sum of the zeroes } \alpha + \beta = \frac{-\text{coefficinet of } x}{\text{coefficient of } x^2} = \frac{-b}{a}$$

$$\text{Product of the zeroes } \alpha \beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{c}{a}$$

Formation of a quadratic polynomial $x^2 - (\alpha + \beta)x + \alpha\beta$

1. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$ and verify the relationship between the zeroes and the coefficients.

$$\text{Soln.: } x^2 + 7x + 10 = x^2 + 5x + 2x + 10$$

$$x(x+5) + 2(x+5) = (x+5)(x+2)$$

$$\text{Let } x+5=0 \text{ or } x+2=0; x=-5 \text{ (or) } x=-2$$

$$\text{Sum of the zeroes} = (-5) + (-2) = -7 = \alpha + \beta$$

$$\text{Product of the zeroes} = (-5) \times (-2) = 10 = \alpha\beta$$

Verification:

$$a=1, b=7, c=10$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{1} = -7$$

$$\alpha\beta = \frac{c}{a} = \frac{10}{1} = 10$$

Home work:

Find the zeroes and verify the relationship between the zeroes and coefficients.

i) $x^2 + 3x + 2$ (iii) $x^2 - 2x - 8$ (v) $6x^2 - 7x - 3$
ii) $4x^2 - 4x + 1$ (iv) $3x^2 - x - 4$

NB : (i) Study the given explanations.

(ii) By heart all the formulae.